

Tumbling of vesicles under shear flow within an advected-field approach

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We study dynamics of vesicles with a viscosity contrast between the interior and exterior, and subjected to a linear shear flow. It is shown that the vesicle, which assumes a steady orientation with respect to the shear flow for $r \equiv \eta_{in}/\eta_{out} < r_c$ (where η_{in} and η_{out} are the internal and external viscosities, and r_c is a critical value which depends upon the swelling factor), undergoes a *tumbling* bifurcation for $r \geq r_c$. Tumbling occurs as a saddle-node bifurcation. We present analytical and numerical results. We develop a powerful method, the advected-field approach. This method allows one to treat several different phenomena with great flexibility.

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Problems in which a viscous fluid interacts with a deformable entity are of paramount importance in the study of blood, polymer solutions, and suspension of liquid droplets. For example, over the last decades considerable efforts have been devoted to the understanding of circulation physiology and suspension rheology [1,2] with the aim of elucidating the mechanisms that govern the human red cell transport both *in vivo* and *in vitro*. Under shear flow red blood cells may align at moderate shear rates. When the same blood is diluted enough with plasma the same cells undergo a tumbling process [3].

In this paper we deal with the tumbling process and develop a powerful method, the advected-field approach. As a starting point for the modeling, we concentrate on the dynamics of a single vesicle made of pure phospholipids. Vesicles are closed membranes, which are suspended in an aqueous solution [4]. They constitute a model system of biological membranes.

One of the major difficulties in studying the interaction between a vesicle and an external hydrodynamic flow lies in the free-boundary character of its vesicle shape. That is, the shape is not given *a priori* but it results from a compromise between flow, bending energy, and various physical constraints. This results in nonlocal and nonlinear equations. Studies allowing for shape evolution of vesicles have been made possible only recently [5–7] with the assumption that the same fluid is inside and outside the vesicle. The red cell problem has several known theoretical treatments, analytical works based on lubrication theory [1] as well as numerical studies (see Ref. [8] for a recent review). The precise analysis of tumbling has not been yet analyzed, either for vesicles or for red cells. The first aim of the present paper is to analyze the tumbling transition for vesicles, which is a simpler system than red cells (which have a cytoskeleton, and the rheology of which is still a matter of debate). Our strategy is to have a full understanding of this system before including further complexity.

We find that if the viscosity inside the vesicle, η_{in} , is different from that outside, η_{out} , then the vesicle undergoes a tumbling bifurcation at $r \equiv \eta_{in}/\eta_{out} = r_c$, where r_c is a dimensionless quantity which depends on the swelling factor (defined below). For $r < r_c$, the vesicle assumes a steady state solution with its long axis making an angle with the shear flow direction. One of the results that emerges from

this analysis is that the tumbling occurs as a saddle-node bifurcation. We account for this bifurcation on the basis of general considerations.

Recent approaches to vesicle dynamics [5–7] have used a boundary integral formulation based on the Green's function technique. Due to the linearity of the Stokes equation, a free space Green's function is easy to compute. For non-Newtonian fluids, such a formalism is not available. In addition, boundary integral methods do not allow for a topology change in a natural way. In order to handle this problem, *ad hoc* rules have been prescribed [9]. In this paper we develop the *advected-field* approach. This method, though it is directly inspired by the phase-field approach (see Ref. [10] for a recent review), poses a challenge, since one must impose a local incompressibility of the local area element of the membrane. Thus, a difference with respect to traditional phase-field approaches lies in the introduction of dynamics along the contour. Note that since the membrane is not a boundary between “true” phases, but rather between the two domains of the same phase, we have felt it more legitimate to call this method the “advected field” (the membrane is advected by the flow).

As is the case with phase-field approaches, the crux of the advected-field (AF) approach is to introduce an auxiliary field $\phi(\mathbf{r}, t)$, which varies continuously, albeit abruptly, from the interior ($\phi = -1$) of the vesicle to the exterior ($\phi = 1$). Thus, instead of treating the membrane, as usual, as a geometrical location on which boundary conditions are to be imposed (this is the *sharp-boundary* method, hereafter referred to as SB), the boundary is the location of a rapid spatial variation of ϕ . The membrane is advected by the flow. Appropriate evolution equations for ϕ are

$$\frac{\partial \phi}{\partial t} = -\mathbf{v} \cdot \nabla \phi + \epsilon_\phi \left(-\frac{\delta E_{intrinsic}}{\delta \phi} + c \epsilon^2 \left| \nabla \phi \right| \right), \quad (1)$$

$$E_{intrinsic}[\phi] = \int \int dx dy \left\{ \frac{1}{4} (1 - \phi^2)^2 + \frac{\epsilon^2}{2} (\nabla \phi)^2 \right\}. \quad (2)$$

Minimization of this function in a one-dimensional geometry with the boundary conditions $\phi(\pm\infty) = \pm 1$ leads to the so-

lution $\phi(r) = \tanh(r/\epsilon\sqrt{2})$. The width of the boundary is $\epsilon\sqrt{2}$. For ease of presentation, we shall confine ourselves to two dimensions. We have checked (see below) that this captures three-dimensional (3D) physical features. The term $E_{intrinsic}$ plays the role of a local restoring force to the desired tanhlike shape. $E_{intrinsic}$ is similar to a Landau-Ginzburg free energy with used coexistence between two ‘‘phases’’ $\phi = -1$ and $\phi = 1$. The $|\nabla\phi|$ term is used here to cure a generic problem due to wall energy, which is nothing but the surface energy. The quantity c is the curvature in the AF spirit, to be defined below. As a vesicle has no surface tension, this term is introduced precisely in order to cancel the wall free energy [11,12].

The next step is to specify the dynamics of the velocity field in the whole domain. For that purpose one must determine the membrane forces. The Helfrich [4] free energy in the AF sense takes the form

$$E_{config} = \frac{\kappa}{2} \int \int dx dy c^2 \frac{|\nabla\phi|}{2} + \int \int dx dy \zeta \frac{|\nabla\phi|}{2}. \quad (3)$$

The curvature field c can easily be expressed in terms of ϕ if we define the normal vector field $\hat{\mathbf{n}}$ and the tangential one $\hat{\mathbf{t}}$ as $\hat{\mathbf{n}} = \nabla\phi/|\nabla\phi|$, $\hat{\mathbf{t}} = \hat{\mathbf{n}} \times \hat{\mathbf{z}}$. This definition corresponds to the choice $d\hat{\mathbf{t}}/ds = +c\hat{\mathbf{n}}$ in curvilinear coordinates. One can easily check that $c = -\nabla \cdot \hat{\mathbf{n}}$. κ is the bending rigidity and $\zeta(\mathbf{r}, t)$ is, in the SB sense, a Lagrange multiplier which enforces local incompressibility of the vesicle area in the course of time. ζ is a more complex field, which cannot be expressed as a simple functional of ϕ (it is history dependent, as we shall see below). The functional derivative of E_{config} provides us with the membrane force,

$$\mathbf{F}_{config} = \left[-\kappa \left\{ \frac{c^3}{2} + \hat{\mathbf{t}} \cdot \nabla (\hat{\mathbf{t}} \cdot \nabla c) \right\} \hat{\mathbf{n}} + \zeta c \hat{\mathbf{n}} + \hat{\mathbf{t}} \cdot \nabla \zeta \hat{\mathbf{t}} \right] \frac{|\nabla\phi|}{2}, \quad (4)$$

which reduces to the two-dimensional (2D) expression in the SB limit [6], when $\epsilon \rightarrow 0$ [11]. Note that in that limit $|\nabla\phi|/2$ coincides with a δ function across the membrane. Once the force is known, we are in a position to write the hydrodynamical equation:

$$\epsilon_v \frac{\partial \mathbf{v}}{\partial t} = \eta \Delta \mathbf{v} - \nabla P + \mathbf{F}_{config}, \quad (5)$$

where ϵ_v is a density scale, which is related to the relaxation time. ϵ_v will be chosen in such a way that inertia is small (the Stokes limit). We could set the left-hand side of Eq. (5) to zero. Then we should have been forced at each time step to solve a static equation by a Newton-Raphson scheme (since the equation is nonlinear due to the dependence of η on ϕ ; see below). It is well known that this procedure does not always lead to convergence (even if a solution does exist, the Newton-Raphson scheme may fail to find it). Imposing a small enough value for ϵ_v allows us to treat the dynamics in an iterative scheme, a straightforward procedure. In any case

the Reynolds number is never exactly equal to zero, but lies typically between 10^{-2} and 10^{-4} in most situations of interest.

The pressure field must be adjusted to ensure incompressibility:

$$\nabla \cdot \mathbf{v} = 0.$$

In the presence of a viscosity contrast between the interior and the exterior of the vesicle, $\eta \Delta \cdot \mathbf{v}$ in the Stokes equation has to be substituted by $\partial_i [\eta (\partial_i v_j + \partial_j v_i)]$ since η is now position-dependent. Note that repeated indices are to be summed over. A simple prescription for η is $\eta = \eta_{out}(1 + \phi)/2 + \eta_{in}(1 - \phi)/2$.

Finally, the ζ field has to be determined. To date no phase-field model has dealt with a field along the tangent of the contour. We introduce the following dynamical law:

$$\frac{\partial \zeta}{\partial t} = -\mathbf{v} \cdot \nabla \zeta + T \hat{\mathbf{t}} \cdot (\hat{\mathbf{t}} \cdot \nabla) \mathbf{v}, \quad (6)$$

where T is a tensionlike constant and $\hat{\mathbf{t}} \cdot (\hat{\mathbf{t}} \cdot \nabla) \mathbf{v}$ represents $\hat{\mathbf{t}} \cdot \partial \mathbf{v} / \partial s$ in the SB limit, which is nothing but the local extension rate of the membrane. With this prescription, ζ is proportional to the local extension of the membrane. Imposing a large enough T enforces a vanishingly small extension. The ζ reaction is confined within the membrane thanks to the δ -like function in Eq. (4).

The boundary position is automatically encoded in the rapid variation of the field ϕ . Besides this advantage, implementing any new effect, such as viscosity contrast, requires no special deal. Finally, any other hydrodynamical constitutive law (e.g., for complex fluids) can be accounted for with no additional conceptual complication.

It remains to be shown formally that in the limit $\epsilon \rightarrow 0$, the AF equations reduce to the SB ones used in Ref. [6]. This is performed by a (singular) formal expansion of Eqs. (1)–(6) [11]. We have checked that the relaxation towards equilibrium shapes [13], as well as the orientation of the vesicle in a shear flow [5] (in 2D and 3D) obtained with the SB method with $\eta_{in} = \eta_{out}$, are captured by the AF approach.

We now address the tumbling problem. We consider that the vesicle is subjected to a linear shear flow given by $v_x = \gamma y$ and $v_y = 0$ (x is the horizontal coordinate in Fig. 1), where γ is the shear rate. We can form five independent parameters. The first trivial one is the viscosity ratio. The effective Reynolds number is defined as $R_e = \epsilon_v \gamma R^2 / \eta_{out}$, where R is the effective radius obtained from $\sqrt{S/\pi}$, where S is the enclosed area. For most simulations we have chosen $R_e \sim 10^{-2}$ so that inertia is small. The second parameter is the ‘‘capillary’’ number associated with the bending mode, $C_b = \eta_{out} \gamma R^3 / \kappa$, and this number is taken to be of order one (meaning that the shape of the vesicle changes under flow on the same time scale as that associated with the flow, as is observed experimentally). The tension field T defines another capillary number $C_t = \eta_{out} \gamma R / T$, which is taken to be very small (typically 10^{-4}) in order to preserve local incompressibility of the membrane on the time scale imposed by the shear flow. Finally, the ‘‘transition’’ width of the advected

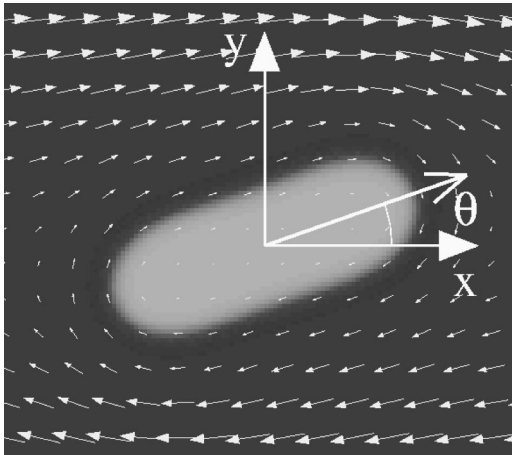


FIG. 1. The vesicle (in gray) with $r=0.75$. Arrows represent the local velocity field. Shear flow is along the x axis.

field ϵ is taken to be small enough in comparison to the vesicle size (typically less than 0.1). The viscosity ratio and the swelling factor have been varied in a quite wide range as shown in Figs. 2 and 3. As far as the other parameters are concerned, we have not yet scanned the whole parameter space, but have confined ourselves to typical realistic values. For example, the vesicle rigidity varies for most phospholipids within a limited range (typically between $20k_B T$ and $40k_B T$, where $k_B T$ is the thermal excitation energy). Within this range the overall tumbling picture is unaffected, though some minor quantitative differences are detected. A systematic scan of the whole parameter space will be performed in the future.

The behavior of a particle in a shear flow is a long-standing problem [14]. Several attempts have been made in the context of red blood cells [15–18]. These authors make several important simplifications. For example, to date none of them took into consideration either the free-boundary character, i.e., that the shape is not given *a priori* (but evolves with hydrodynamical constraints) or the dynamics of

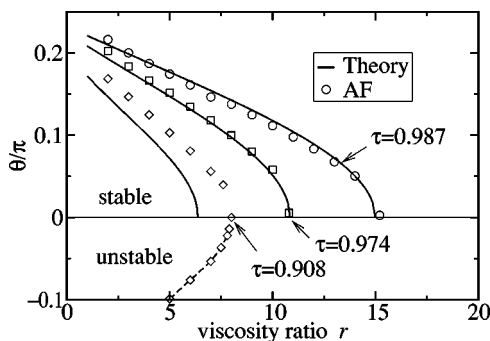


FIG. 2. Stationary angle θ vs the viscosity ratio for several swelling factors τ . The full lines correspond to the fixed shape theory [16], while the symbols are the simulation data. A finite size scaling analysis has been performed to cope with finite size effects. The dashed line corresponds to the unstable branch for $\tau=0.908$ in the AF approach. Agreement is good only for $r>0.9$, while below more than 100% disagreement is visible. This is may be due to an improper account of tank treading.

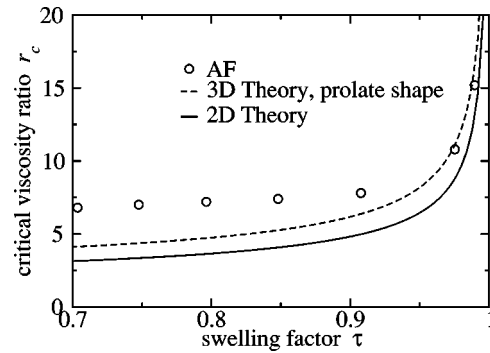


FIG. 3. Critical viscosity ratio r_c as a function of τ . This figure illustrates the disagreement between the theory of Ref. [16] and simulation data for $\tau<0.95$. A 3D branch for a prolate shape is also shown. Amazingly, r_c reaches a plateau at low values of τ , which corresponds to $r_c \approx 7$.

the tension field ζ . This sets serious limitations on the use of their results for deformable entities, such as erythrocytes, or vesicles. In the present paper an arbitrary shape evolution, compatible with forces entering into play, are allowed. Recently in Ref. [8] a boundary integral method for capsules has been used. It was noted that tumbling may occur after a certain value of the viscosity contrast. However, the precise nature of tumbling and its generic character and how this is connected with dissipation have remained unanswered questions. These questions are analyzed in this paper.

We shall merely report on the physical results, leaving the numerical scheme with various details for a future publication [11]. Our notations together with the nature of the velocity fields are shown in Fig. 1. Under shear flow, dynamics reveal the following behaviors. At a small enough viscosity contrast $r \equiv \eta_{in} / \eta_{out} < r_c$, the vesicle assumes (after transients) a steady-state orientation with angle θ . Note that the membrane undergoes a *tank-treading* motion. The angle depends both on the aspect ratio [or swelling factor τ , defined as $\tau = 4\pi S / P^2$, where S is the internal area, P the perimeter (2D); for a circle $\tau=1$] and r . For a given τ , the angle θ decreases on increasing r until it reaches 0 at r_c (Fig. 2). The same scenario holds for a given r when decreasing τ . For $r > r_c$, the vesicle tumbles with a complex tank-treading motion. We have noted that the tumbling transition is sensitive to the boundary effects: for a small box tumbling is delayed due to the increase of dissipation with the walls. We have increased the size of the box in numerical simulations. So far, when the size exceeds about three times the vesicle size, only minor changes are observed. This must be subject to caution, however, since in 2D the wall effect might still be detectable even at long distances and thus some quantitative “drift” of the results may be revealed. Extensive quantitative studies are currently under investigation.

Tumbling occurs as a saddle-node bifurcation. That is, close to r_c the steady-state branch $\theta(r)$ (corresponding to a saddle in the terminology of bifurcations) ceases to exist for $r > r_c$, whereby it merges with an unstable branch (node) as shown in Fig. 2. In contrast to usual instabilities, for which the stable branch turns into an unstable or metastable branch above r_c , no steady-state solution exists in the present case

above r_c . Tumbling is thus not an instability in the usual sense.

Given these results, it is important that one understand this behavior on the basis of general considerations. Let $\hat{\mathbf{m}}$ be a unit vector, pointing along the main axis of the vesicle, and let us treat the vesicle as a local “dipole” in a fluid (with no extent on scales of the system). From Newton’s law and kinematics, one can write an evolution equation [11] for $\hat{\mathbf{m}}$ as follows:

$$\frac{\partial \hat{\mathbf{m}}}{\partial t} = \hat{\mathbf{m}} \cdot \boldsymbol{\omega} + \alpha \hat{\mathbf{m}} \cdot \boldsymbol{\Lambda} + \beta \hat{\mathbf{m}}. \quad (7)$$

The left-hand side is the rate of rotation. $\boldsymbol{\omega} = (\nabla \cdot \mathbf{v} - \nabla \cdot \mathbf{v}^T)/2$ is the vorticity tensor. This term describes the rigid body rotation in the velocity field. The last two terms are the viscous force $\boldsymbol{\Lambda}$, the symmetric part of the stress tensor (the first contribution to the force is linear in the velocity gradient), and β is a Lagrange multiplier enforcing a constant length for $\hat{\mathbf{m}}$. After a scalar product of Eq. (7) with $\hat{\mathbf{m}}$ one gets $\beta = -\alpha \hat{\mathbf{m}} \cdot \hat{\mathbf{m}} \cdot \boldsymbol{\Lambda}$. α is a parameter specifying the geometry of the body, and for an ellipsoid it should coincide with $\alpha = [(a/b)^2 - 1]/[(a/b)^2 + 1]$ (a and b are the length of the long and short axes of the ellipse) as found from a direct calculation by Jeffery [14] for a rigid body. The most serious point is to determine the stress tensor, which is a formidable task in general. Neglecting backflow, we can simply write $\mathbf{v} = \gamma y \hat{\mathbf{x}}$. Writing $\hat{\mathbf{m}} = [\cos(\theta), \sin(\theta)]$, one obtains

$$\gamma^{-1} \frac{\partial \theta}{\partial t} = -1 + \xi \cos(2\theta). \quad (8)$$

From this expression, a steady angle can exist if $\xi > 1$; a value of 1 would correspond to $\theta = 0$, the critical angle. For a rigid ellipse, we have $\xi = [(a/b)^2 - 1]/[(a/b)^2 + 1] < 1$ and, therefore, no steady-state solution exists; tumbling then always occurs. If the tank-treading flow [11] motion is taken into account (which depends on η_{in} and η_{out}), ξ is now a function of the viscosity ratio r [18] and it may be either greater or smaller than unity. There is then a critical value r_c for which $\xi = 1$. After [18] it happens that the steady solution exists for $r \leq r_c$. Close to r_c , but below, $\xi - 1 \sim A(r_c - r)$, where the constant $A > 0$ since $\xi > 1$ (steady regime). We thus have $\gamma^{-1} \partial \theta / \partial t = A(r_c - r) - \theta^2/2$, where two steady-state branches $\theta_{\pm} \sim \pm \sqrt{r_c - r}$ are found. For $r > r_c$, these branches cease to exist. Linear stability analysis around the steady-state solutions easily shows that θ_+ is stable while θ_- is unstable.

As stated above, a rigid body would tumble without threshold. For vesicles we have demonstrated the existence of a threshold value: the viscosity inside must be greater than that outside (a ratio, which depends on the swelling factor). Liquid droplets [9] with a viscosity is approximately four times smaller than that of the ambient fluid either assume a steady-state shape with the long axis making a certain angle with the flow (if the shear rate is small enough) or break up into smaller droplets (if the shear rate exceeds a certain value). For a viscosity contrast larger than about 4,

an ellipsoidal droplet is obtained, aligned in the flow direction, that cannot be broken at any shear rate. In any of these cases, tumbling occurs. Tumbling is present within vesicles not only due to viscosity contrast, but also due to the fact that the contour of the vesicle cannot be stretched (in contrast to droplets). Thus the local incompressibility dealt with here is essential for tumbling.

Finally, inspection of the dissipation

$$\dot{E}_{hydro} = \int \frac{\eta}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)^2 dV \quad (9)$$

generated by the AF dynamics reveals several interesting features that are briefly presented. The global dissipation decreases as r is increased (at constant τ). So far, we have found that dissipation exhibits a minimum [11] for $r \approx r_c$. *A priori* there is no reason that the general Rayleigh principle applies, as boundaries are not fixed [19]. It may be noted that in the blood, red cells under shear become elongated and oriented in the flow, conferring on the blood a shear thinning property [3] (collapse of the actual viscosity on increasing the shear rate). When the cell concentration is low, cells tumble precisely in order to reduce the dissipation. Thus tumbling is an alternative way for cells to lower the dissipation. We have found that the dissipation inside the vesicle is negligible in comparison with that outside. This result can easily be understood since for a circular shape (spherical in 3D) tank treading corresponds to a global rotation of the vesicle and the sphere moves as a rigid body inside. Thus no dissipation occurs. The only internal dissipation results from a deviation from sphericity. From these considerations and Eq. (9), one expects the following scaling law for the dissipation inside the vesicle to hold:

$$\dot{E}_{in} \propto \eta_{in} v_{tank}^2 (1 - \tau)^\alpha, \quad (10)$$

where v_{tank} is the tank-treading velocity. The exponent α must be equal to one since we expect no critical behavior (in the sense of instabilities) when $\tau \rightarrow 1$. Numerical evaluation of α confirms this result within good accuracy (actually, $\alpha \approx 0.9$).

In summary, we have developed an advected-field approach, which opens an avenue of promising powerful applications for the study of the interaction of deformable entities with flows. In light of this work, matters such as droplet-droplet collision and fusion, bud emission in vesicles, and breakup of droplets under shear, in both Newtonian and non-Newtonian fluids, become quite accessible. Besides the problem of vesicles, where boundary integral methods have been used [5–7], several works on droplet breakup have been performed under shear (for a review, see Ref. [9]). In order to allow for a topology change, a quite natural problem during droplet shearing, some rules must be invented within the integral formulation, and often in an *ad hoc* manner [9] (not to mention the fact that the increase of the number of interfaces occurring, for example, after breakup, must be dealt with explicitly; see Ref. [20] and Refs. therein). These are front-tracking methods, contrary to our method, which treats very naturally the change of topology, and does not

need to follow any interface. In addition, the proliferation of interfaces does not add new complexity within the AF approach. Note that volume-of-fluid interface tracking [21], or level set methods [22] could also be alternatively used, but to date none of these methods has been used for vesicles, and only qualitative results on droplet breakup are available.

In conclusion, we have demonstrated the power of the advected-field method in the context of tumbling. Our results differ from those obtained with nondeformable entities. Erythrocytes have the ability to undergo large deformations due to their membrane elasticity. This effect will be incorporated in the near future in the full 3D treatment. In order to shed light on the importance of membrane elasticity, this work should also stimulate more experiments on vesicles filled with a more viscous fluid. Another point is that although our method allows naturally for vesicle budding, for example, the mechanism that leads to this process is not included within the energy given here. In addition, during breaking, microscopic considerations (such as phospholipids interactions) should play a decisive role, and must thus be included in order to treat a budding process. Another important point is that our membrane energy is taken here to be the simplest one possible, and has thus disregarded several features that may become important in more realistic situations.

For example, we neglected the spontaneous curvature effect, as well as the area difference between the two monolayers [23]. The presence of a spontaneous curvature in the model is equivalent to an effective surface tension (such as our field ζ) plus a term $\int cds$ where c is the curvature. In 2D this is irrelevant, while in 3D it plays a role, and it can be included straightforwardly, since in our model integrals on curvature are already evaluated. Another important effect is the area difference between the two monolayers [23]. As shown in Ref. [23], the contribution of the area difference can be written as an integral over the curvature, and for the same reasons evoked above, this can be included quite naturally in the AF method. A more serious point to be emphasized is the incorporation of the fact that the two monolayers forming the membrane may slide with respect to each other, inducing thus a velocity discontinuity at the membrane, resulting in dissipation within the membrane [24,25]. This question requires further investigation.

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